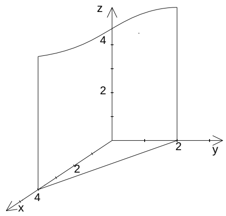
Trabajo Práctico Nº 1: Funciones de varias variables

Representación de superficies:

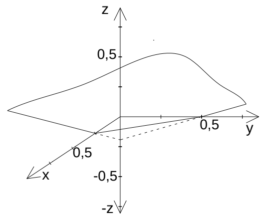
1. a) Plano



Trazas: Plano "xy": $y = -\frac{1}{2}x + 2$; plano "xz": $x = 4$; plano "yz": $y = 2$.

Intersecciones con los ejes coordenados: eje x: $x = 4$; eje y: $y = 2$; eje z: no existe.

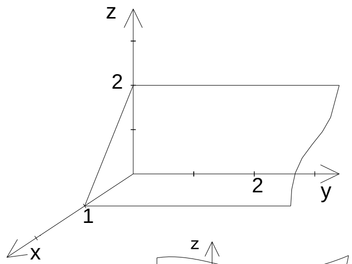
b) Plano



Trazas: Plano "xy": $y = -2x + 1/2$; plano "xz": $z = \frac{4}{3}x - \frac{1}{3}$; plano "yz": $z = \frac{2}{3}y - \frac{1}{3}$

Intersecciones con los ejes coordenados: eje x: $x = {}^{1}/_{4}$; eje y: $y = {}^{1}/_{2}$; eje z: $-{}^{1}/_{3}$.

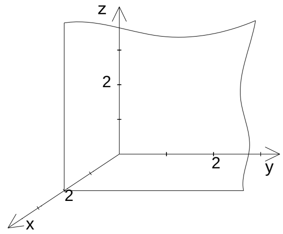
c)Plano paralelo al eje y: $2x + z = 2$



Trazas: Plano "xy": $x = 1$; plano "xz": $z = -2x + 2$; plano "yz": $z = 2$.

Intersecciones con los ejes coordenados: eje x: $x = 1$; eje y: no tiene; eje z: $z = 2$.

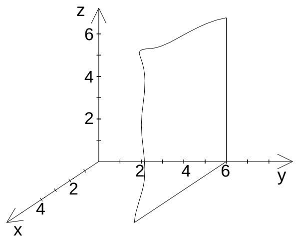
d) Plano



Trazas: Plano "xy": $x = 2$; plano "xz": $x = 2$; plano "yz": no tiene.

Intersecciones con los ejes coordenados: eje x: $x = 2$; eje y: no tiene; eje z: no tiene.

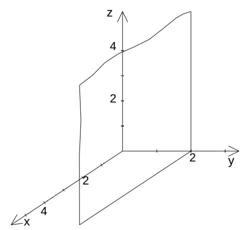
e) Plano



Trazas: Plano "xy": $y = 6$; plano "xz": no tiene; plano "yz": $y = 6$.

Intersecciones con los ejes coordenados: eje x: no tiene; eje y: $y = 6$; eje z: no tiene.

f) Ecuación plano paralelo al plano xz: $y = 2$

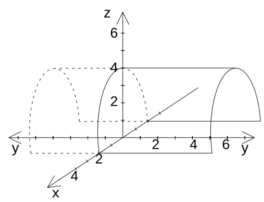


Trazas: Plano "xy": $y = 2$; plano "xz": no tiene; plano "yz": $y = 2$.

Intersecciones con los ejes coordenados: eje x: no tiene; eje y: $y = 2$; eje z: no tiene.

g) Ecuación plano coordenado "xy": $z = 0$; plano coordenado "xz": $y = 0$; plano coordenado "yz": $x = 0$.

h) Superficie cilindrica elíptica



Trazas: Plano "xy":$x = -2 \vee x = 2$;

plano "xz":$\frac{x^2}{4} + \frac{z^2}{16} = 1 \wedge z \geq 0$; plano "yz":$z = 4$.

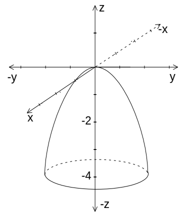
Intersecciones con los ejes coordenados:

eje x: $x = -2 \vee x = 2$; eje y: no tiene; eje z: $z = 4$.

i) Paraboloide elíptico

Trazas: Plano "xy": $4x^2 + y^2 = 0$; plano "xz": $z = -2x^2$; plano "yz": $z = -\frac{1}{2}y^2$

Intersecciones con los ejes coordenados: eje x: $x = 0$; eje y: $y = 0$; eje z: $z = 0$.

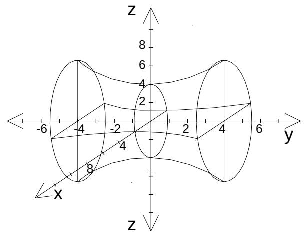


j) Hiperboloide de 1 hoja

Trazas: Plano "xy": $\frac{x^2}{4} - \frac{y^2}{9} = 1$; plano "xz": $\frac{x^2}{4} + \frac{z^2}{16} = 1$; plano "yz": $-\frac{y^2}{9} + \frac{z^2}{16} = 1$

Intersecciones con los ejes coordenados:

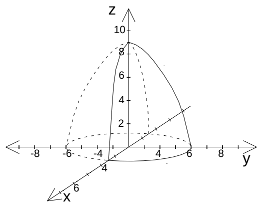
eje x: $x = -2 \vee x = 2$; eje y: no tiene; eje z: $z = -4 \vee z = 4$.



k) Paraboloide elíptico

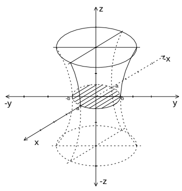
Trazas: Plano "xy": $\frac{x^2}{9} + \frac{y^2}{36} = 1$; plano "xz": $z = -x^2 + 9$; plano "yz": $z = -\frac{y^2}{4} + 9$

Intersecciones con los ejes coordenados: eje x: $x = -3 \vee x = 3$; eje y: $y = -6 \vee y = 6$; eje z: $z = 9$.

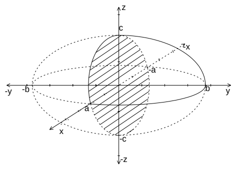


2)

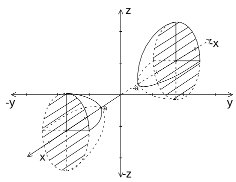
a) Hiperboloide de 1 hoja (no corta al eje "z")



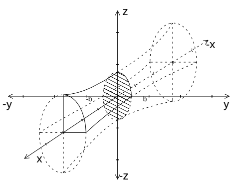
b) Elipsoide



c) Hiperboloide de 2 hojas (no corta al eje "y")

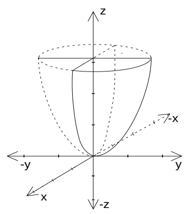


d) Hiperboloide de 1 hoja (no corta al eje "x")

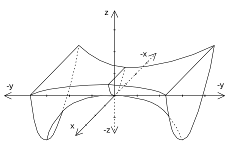


e) No tiene representación

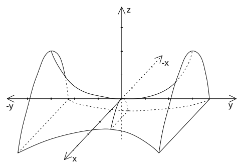
f) Paraboloide elíptico



g) Paraboloide hiperbólico

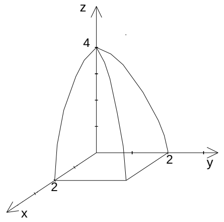


h) Paraboloide hiperbólico

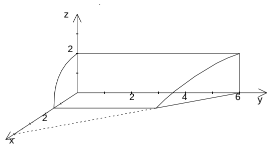


3)

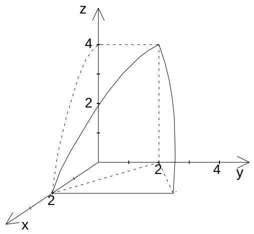
a)



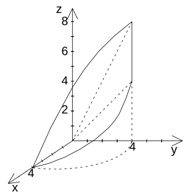
b)



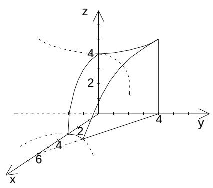
c)



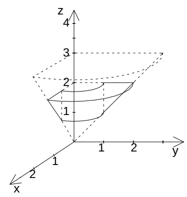
d)



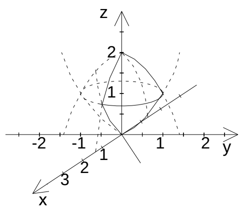
e)



f)



g)



h)$y^2 + z^2 = -4x$. Superficie: paraboloide de revolución.

Funciones de dos variables

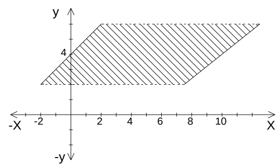
1) a) 0 b) -6 c) 7 d) -9

2) a) -1 b) 0 c) 0 d) 0 e) $\frac{a^2}{a^4-1}$

3) a) La función no está definida.

b) La función está definida.

4) **A)** a)



b) $A' = \{(x;y)/|x-y| \leq 4 \wedge |y-4| \leq 2\}$

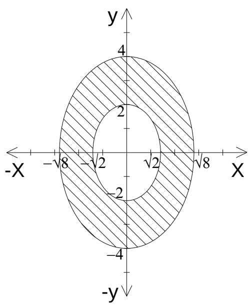
c) $A_i = \{(x;y)/|x-y| < 4 \wedge |y-4| < 2\}$

d) No es conjunto abierto ni cerrado.

e) $A_f = \{(x;y)/(|x-y| = 4 \wedge |y-4| < 2) \vee (|x-y| \leq 4 \wedge |y-4| = 2)\}$

f) Conjunto Conexo.

B) a)



b) $B' = B$

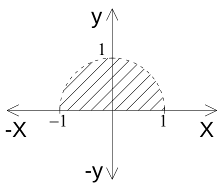
c) $B_i = \{(x;y)/2 < x^2 + \frac{y^2}{2} < 8\}$

d) El conjunto es cerrado.

e) $B_f = \{(x;y)/\frac{x^2}{2} + \frac{y^2}{4} = 1 \vee \frac{x^2}{8} + \frac{y^2}{16} = 1\}$

f) Conjunto conexo.

C) a)



b) $C' = \{(x;y)/x^2 + y^2 \leq 1 \wedge y \geq 0\}$

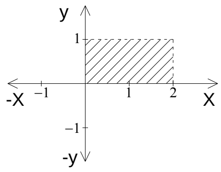
c) $C_i = \{(x;y)/x^2 + y^2 < 1 \wedge y > 0\}$

d) No es conjunto abierto ni cerrado.

e) $C_f = \{(x;y)/(x^2 + y^2 = 1 \wedge y > 0) \vee (x^2 + y^2 \leq 1 \wedge y = 0)\}$

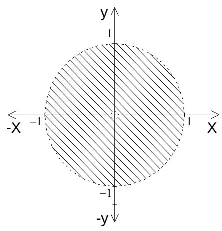
f) Conjunto conexo.

D) a)



b) $D' = \{(x;y)/0 \leq x \leq 2 \wedge 0 \leq y \leq 1\}$
c) $D_i = \{(x;y)/0 < x < 2 \wedge 0 < y < 1\}$
d) No es conjunto abierto ni cerrado.
e) $D_f = \{(x;y)/[(x = 0 \vee x = 2) \wedge 0 < y < 1] \vee [0 \leq x \leq 2 \wedge (y = 0 \vee y = 1)]\}$
f) Conjunto conexo.

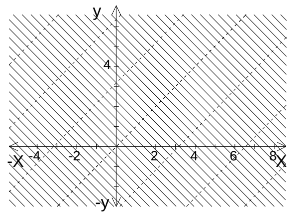
E) a)
$E = \{(x;y)/\ x^2 + y^2 > 0\} \cap \{(x;y)/\ x^2 + y^2 < 1\}$



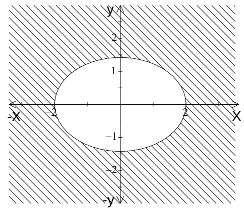
b) $E' = \{(x;y)/\ x^2 + y^2 \geq 0 \wedge\ x^2 + y^2 \leq 1\}$
c) $E_i = \{(x;y)/\ x^2 + y^2 > 0 \wedge\ x^2 + y^2 < 1\}$
d) El conjunto es abierto.
e) $E_f = \{(x;y)/\ x^2 + y^2 = 0\ \vee x^2 + y^2 = 1\}$
f) Conjunto conexo.
5. a) $A = \{(x;y)/y \geq x^2 \wedge y \leq x + 2\}$
b) $B = \{(x;y)/\frac{x^2}{36} + \frac{y^2}{16} \leq 1 \wedge x^2 + y^2 \geq 1\}$

Dominio e imagen de funciones de dos variables

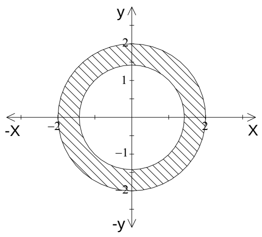
a) $Dom = \{(x;y)/y \neq x + k\pi, k \in \mathbb{Z}\}$;
$Rg = (-\infty; -1] \cup [1; \infty)$.



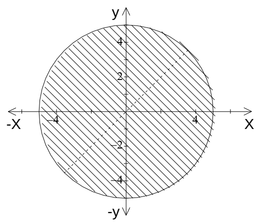
b) $Dom = \{(x;y)/\frac{x^2}{4} + \frac{y^2}{2} \geq 1\}$;
$Rg = \mathbb{R}_0^+$.



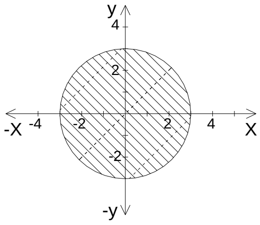
c) $Dom = \{(x;y)/2 \leq x^2 + y^2 \leq 4\}$;
$Rg = [0; \frac{\pi}{2}]$



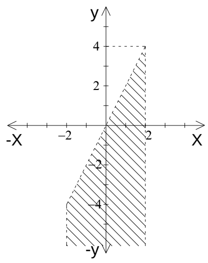
d) $Dom = \{(x;y)/x^2 + y^2 \leq 25 \wedge x \neq y\}$;
$Rg = \mathbb{R}$.



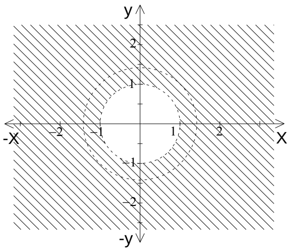
e) $Dom = \{(x;y)/x^2 + y^2 \leq 9 \wedge y \neq x + k\pi, k \in \mathbb{Z}\}$;
$Rg = \mathbb{R}$.



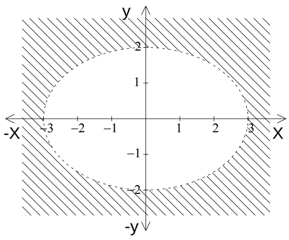
f) $Dom = \{(x;y)/y < 2x \wedge x^2 < 4\}$;
$Rg = \mathbb{R}$.



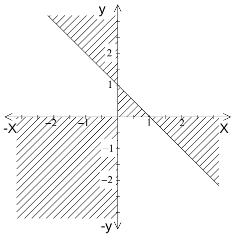
g) $Dom = \{(x;y)/x^2 + y^2 \neq 2 \wedge x^2 + y^2 > 1\}$;
$Rg = \mathbb{R} - \{0\}$.



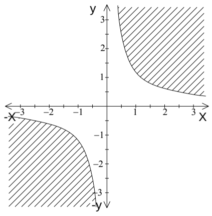
h) $Dom = \{(x;y)/\frac{x^2}{9} + \frac{y^2}{4} > 1\}$;
$Rg = (0;\infty)$.



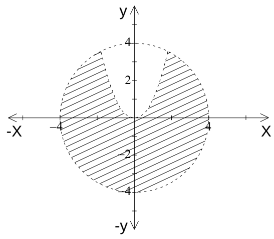
i) $Dom = \{(x;y)/(xy \geq 0 \wedge y \leq -x+1) \vee (xy \leq 0 \wedge y \geq -x+1)\}$; $Rg = \mathbb{R}_0^+$.



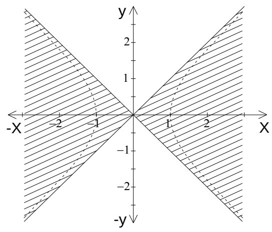
j) $Dom = \{(x;y)/xy \geq 5\}$; $Rg = \mathbb{R}_0^+$.



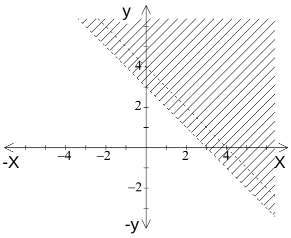
k) $Dom = \{(x;y)/y < x^2 \wedge x^2 + y^2 < 16\}$;
$Rg = \mathbb{R}$.



l) $Dom = \{(x;y)/x^2 - y^2 \neq 1 \wedge x^2 > y^2\}$;
$Rg = \mathbb{R} - \{0\}$.

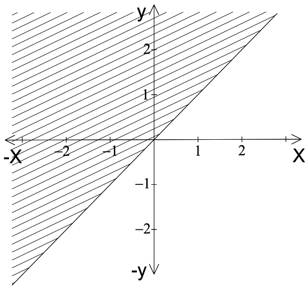


m) $Dom = \{(x;y)/y \neq -x+4 \wedge y > -x+3\}$;
$Rg = \mathbb{R} - \{0\}$.

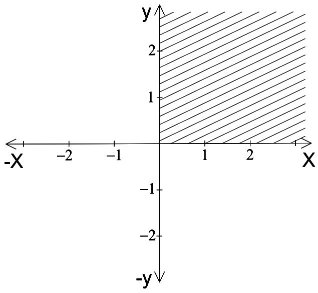


2)

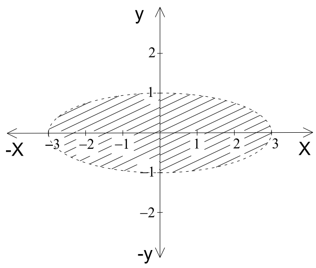
a) $Dom = \{(x;y)/y \geq -x\}$



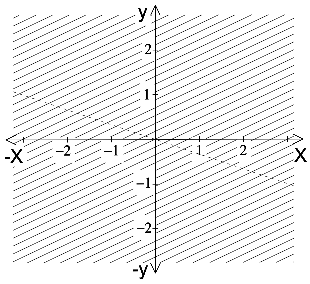
b) $Dom = \{(x;y)/x \geq 0 \wedge y \geq 0\}$



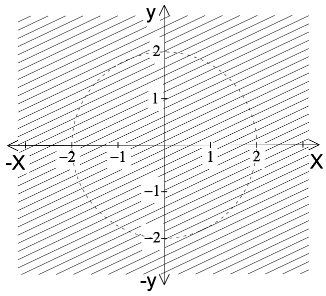
c) $Dom = \{(x; y)/ \frac{x^2}{9} + y^2 < 1\}$



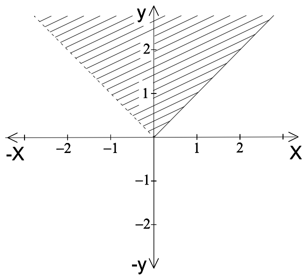
d) $Dom = \{(x; y)/y \neq -\frac{x}{3}\}$



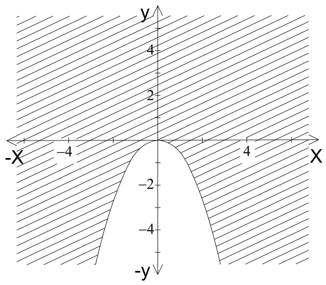
e) $Dom = \{(x; y)/x^2 + y^2 \neq 4\}$



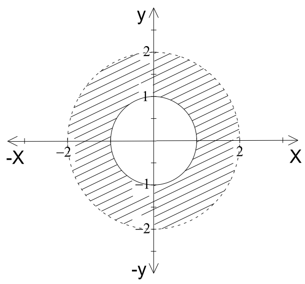
f) $Dom = \{(x; y)/y \geq x \wedge y > -x\}$



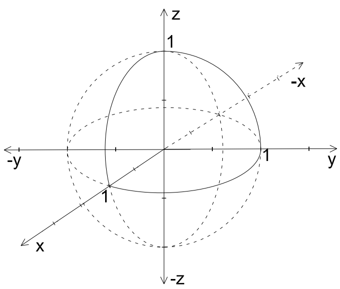
g) $Dom = \{(x; y)/y \geq -x^2\}$



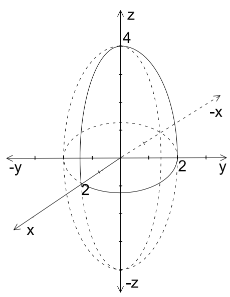
h) $Dom = \{(x; y)/x^2 + y^2 \geq 1 \wedge x^2 + y^2 < 4\}$



i) $Dom = \{(x; y; z)/x^2 + y^2 + z^2 \leq 1\})$



j) $Dom = \{(x; y; z)/ \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} < 1\}$



3) a) 0; b) 1; c) $Dom = \{(x; y)/y > -x + 1\}$; d) $Img = \mathbb{R}$

4) a) 1; b) $Dom = \mathbb{R}^2$; c) $Img = \mathbb{R}^+$;

5) a) $\sqrt{11}$; b) $Dom = \{(x; y)/ \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$; c) $Img = [0; 6]$

6) a) 0; b) $Dom = \{(x; y)/z > -x + y\}$; c) $Img = \mathbb{R}$

7) a) $1/5$; b) $Dom = \{(x; y)/x^2 + y^2 + z^2 > 1\}$; c) $Img = \mathbb{R}^+$

Curvas de nivel

1.

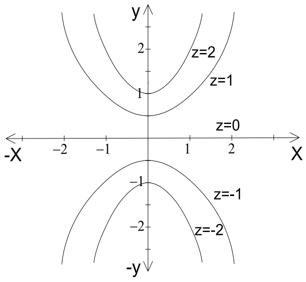
a) $z = -2 \rightarrow y = -\frac{1}{2}x^2 - \frac{1}{2}$

$z = -1 \rightarrow y = -x^2 - 1$

$z = 0 \rightarrow x = 0$

$z = 1 \rightarrow y = x^2 + 1$

$z = 2 \rightarrow y = \frac{1}{2}x^2 + \frac{1}{2}$



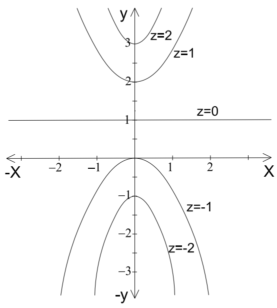
b) $z = -2 \rightarrow y = -2x^2 - 1$

$z = -1 \rightarrow y = -x^2$

$z = 0 \rightarrow y = 1$

$z = 1 \rightarrow y = x^2 + 2$

$z = 2 \rightarrow y = 2x^2 + 3$



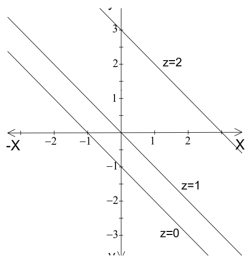
c) $z = -2 \rightarrow \nexists\ curva\ de\ nivel$

$z = -1 \rightarrow y = \nexists\ curva\ de\ nivel$

$z = 0 \rightarrow y = -x - 1$

$z = 1 \rightarrow y = -x$

$z = 2 \rightarrow y = -x + 3$



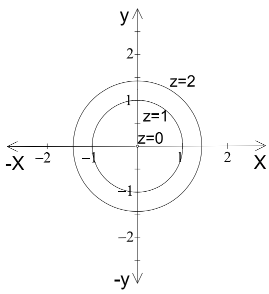
d) $z = -2 \rightarrow \nexists\ curva\ de\ nivel$

$z = -1 \rightarrow \nexists\ curva\ de\ nivel$

$z = 0 \rightarrow x^2 + y^2 = 0$

$z = 1 \rightarrow x^2 + y^2 = 1$

$z = 2 \rightarrow x^2 + y^2 = 2$



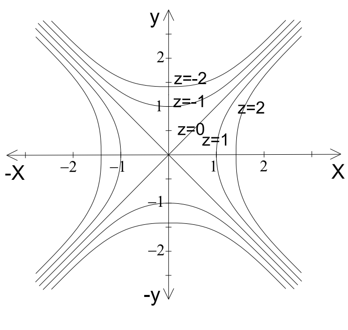
e) $z = -2 \rightarrow \frac{-x^2}{2} + \frac{y^2}{2} = 1$

$z = -1 \rightarrow -x^2 + y^2 = 1$

$z = 0 \rightarrow y = \pm x$

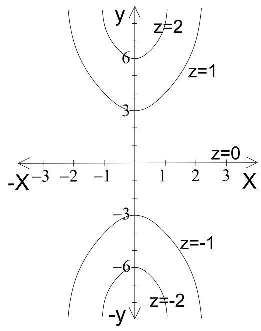
$z = 1 \rightarrow x^2 - y^2 = 1$

$z = 2 \rightarrow \frac{x^2}{2} - \frac{y^2}{2} = 1$

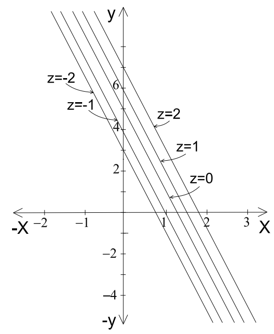


2.

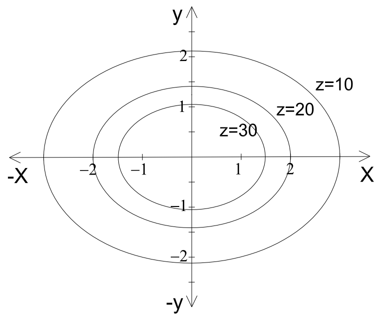
a) $z=-2 \rightarrow y=-2x^2-6$
$z=-1 \rightarrow y=-x^2-3$
$z=0 \rightarrow y=0$
$z=1 \rightarrow y=x^2+3$
$z=2 \rightarrow y=2x^2+6$



b)
$z=-2 \rightarrow y=-4x+3$
$z=-1 \rightarrow y=-4x+4$
$z=0 \rightarrow y=-4x+5$
$z=1 \rightarrow y=-4x+6$
$z=2 \rightarrow y=-4x+7$



3. $z=T(x;y)$
$z=10 \rightarrow \frac{x^2}{9}+\frac{y^2}{9/2}=1$
$z=20 \rightarrow \frac{x^2}{4}+\frac{y^2}{2}=1$
$z=30 \rightarrow \frac{x^2}{7/3}+\frac{y^2}{7/6}=1$



4. $x^2+y^2=r^2-(\frac{c}{v})^2$

5. Ec. a y gráf. I; Ec. b y gráf. III; Ec. c y graf. II.

<u>TRABAJO PRACTICO Nº 2: Límites</u>

Límites

$L=\lim_{(x,y)\to(x_1,y_1)} f(x,y)$; $L_1=\lim_{x\to x_1}\lim_{y\to y_1} f(x,y)$; $L_2=\lim_{y\to y_1}\lim_{x\to x_1} f(x,y)$; $L_r=\lim_{x\to x_1}\lim_{y\to g(x)} f(x,y)$.

1) $L=\nexists$; $L_1=1/2$; $L_2=-1/2$. 2) $L=\nexists$; $L_1=-\frac{2}{3}$; $L_2=-1/2$
3) $L=\nexists$; $L_1=1$; $L_2=-3/2$. 4) $L=\nexists$; $L_1=0$; $L_2=0$
5) $L=3$; $L_1=3$; $L_2=3$. 6) $L=0$; $L_1=0$; $L_2=0$
7) $L=\nexists$; $L_1=0$; $L_2=0$; sobre $y=x$: $L_r=1$
8) $L=\nexists$; $L_1=3$; $L_2=1$. 9) $L=0$; $L_1=0$; $L_2=0$
10) $L=-1$; $L_1=-1$; $L_2=-1$. 11) $L=0$; $L_1=0$; $L_2=0$
12) $L=\nexists$; $L_1=0$; $L_2=0$; sobre $y=x$: $L_r=1/2$
13) $L=0$;$L_1=0$; $L_2=0$
14) $L=\nexists$; $L_1=0$; $L_2=0$; sobre $y=x^2$: $L_r=1/2$
15) $L=1$; $L_1=1$; $L_2=1$. 16) $L=3/2$; $L_1=3/2$; $L_2=3/2$
17) $L=\nexists$; $L_1=3/4$; $L_2=-2/5$; sobre $y=4x$: $L_r=-11/16$
18) $L=0$; $L_1=0$; $L_2=0$

Continuidad

1) a) $f(0;0)=3$; $L=0$; $f(0;0)\neq L \Rightarrow f(x;y)$ es discontínua en $(0;0)$.
b) $f(0;0)=0$; $\nexists L \Rightarrow f(x;y)$ es discontínua en $(0;0)$.
c) $f(0;0)=0$; $L=0$; $f(0;0)=L \Rightarrow f(x;y)$ es contínua en $(0;0)$.
2) a) Contínua. b) Discontínua. c) Discontínua. d) Contínua. e) Discontínua.
3) a) $\{(x;y)/xy\neq 0\}$; b) $\{(x;y)/y<x\}$; c) $\{(x;y)/\frac{x^2}{4}-\frac{y^2}{4}<1\}$
d) $\{(x;y)/(x+y)^2\neq \pi/2+k\pi; k\in\mathbb{Z}\}$; e) $\{(x;y)/y\neq -3/5\,x\}$
f) $\{(x;y)/y\neq 0\}$
4) a) No se puede redefinir la función porque esta no posee límite; b) $f(0;0)=0$;
c) $f(0;0)=2/5$

<u>Trabajo Práctico Nº 3: Derivadas y diferenciales primeras</u>

Derivadas parciales

a) $F'_x(1;2)=48$; $F'_y(1;2)=36$. b) $F'_x(2;1)=8$; $F'_y(2;1)=-8$. c) $F'_x(0;0)=1$; $F'_y(0;0)=1$.
d) $F'_x(2;3)=\sqrt{5}/10$; $F'_y(2;3)=\sqrt{5}/10$. e) $F'_x(2;2)=6$; $F'_y(2;2)=15$
f) $F'_x(1;1)=\sqrt{2}/2$; $F'_y(1;1)=\sqrt{2}/2$
2) a) $F'_x=e^{x-y^2}$; $F'_y=-2ye^{x-y^2}$
b) $F'_x=\frac{3}{2(3x-y)\sqrt{\ln(3x-y)}}$; $F'_y=\frac{-1}{2(3x-y)\sqrt{\ln(3x-y)}}$
c) $F'_x=\frac{y}{3\sqrt[3]{x^2y}}sen(xy)+y\sqrt[3]{xy^2}cos(xy)$; $F'_y=\frac{2x}{3\sqrt[3]{x^2y}}sen(xy)+x\sqrt[3]{xy^2}cos(xy)$
d) $F'_x=2.4^yxy+\frac{1}{\sqrt{y^2-x^2}}$; $F'_y=x^24^y(y.ln4+1)-\frac{x}{y.\sqrt{y^2-x^2}}$
e) $F'_x=\frac{xy^2e^{xy}\,[x^2y+x(2y^2+1)+4y]}{(x+2y)^2}$; $F'_y=\frac{x^2ye^{xy}\,(x^2y+2x(y^2+1)+2y)}{(x+2y)^2}$
f) $F'_x=\frac{x[2(x^2+y^2)\text{arctg}(y/x)-xy]}{y^2(x^2+y^2)}$; $F'_y=-\frac{x^2[-2(x^2+y^2)\text{arctg}(x/y)+xy]}{y^3(x^2+y^2)}$
g) $F'_x=\frac{\left(\frac{2xy}{1+x^4}+\frac{1}{x}\right)(xy^2+1)-[y.\text{arctg}(x^2)+\ln(2x)]y^2}{(xy^2+1)^2}$
$F'_y=-\frac{(xy^2-1)\text{arctg}(x^2)+2xy\ln(2x)}{(xy^2+1)^2}$
h) $F'_x=\frac{3xe^{xy}[(2+yx)(2y^2+x)-x]}{(2y^2+x)^2}$; $F'_y=\frac{3x^2e^{xy}(x^2+2xy^2-4y)}{(2y^2+x)^2}$
i) $F'_x=e^{xy}[y.\tan\left(\frac{x}{y}\right)+\frac{1}{y.\cos^2\left(\frac{x}{y}\right)}]$; $F'_y=e^{xy}[x.\tan\left(\frac{x}{y}\right)-\frac{x}{y^2.\cos^2\left(\frac{x}{y}\right)}]$
j) $F'_x=\frac{2y^2e^{xy}(xy-y^2-1)}{(x-y)^2}$; $F'_y=\frac{2ye^{xy}[(2+yx)(x-y)+y]}{(x-y)^2}$
$F'_x(2;1)=0$; $F'_y(2;1)=10e^2$

3) a) $x.z'_x+y.z'_y=x.\left[\frac{\sqrt{x}.\cos\left(\frac{x}{y}\right)}{y}+\frac{\text{sen}\left(\frac{x}{y}\right)}{2\sqrt{x}}\right]+y.\frac{-\sqrt[2]{x^3}.\cos\left(\frac{x}{y}\right)}{y^2}=\frac{\sqrt[2]{x^3}.\cos\left(\frac{x}{y}\right)}{y}+\frac{\sqrt{x}.\text{sen}\left(\frac{x}{y}\right)}{2}-\frac{\sqrt[2]{x^3}.\cos\left(\frac{x}{y}\right)}{y}=\frac{\sqrt{x}.\text{sen}\left(\frac{x}{y}\right)}{2}=\frac{z}{2}$

b) $x.z'_x+y.z'_y=x.\left[\left(2x+\frac{y^3}{x^2}\right).sen\left(\frac{y}{x}\right)-y.\cos\left(\frac{y}{x}\right)\right]+y.[(x+2y).\cos\left(\frac{y}{x}\right)-\frac{y^2.sen\left(\frac{y}{x}\right)}{x}]=\frac{(2x^3+y^3).sen\left(\frac{y}{x}\right)}{x}-xy.\cos\left(\frac{y}{x}\right)+y(x+2y).\cos\left(\frac{y}{x}\right)-\frac{y^3.sen\left(\frac{y}{x}\right)}{x}=2x^2.sen\left(\frac{y}{x}\right)+\frac{y^3.sen\left(\frac{y}{x}\right)}{x}-xy.\cos\left(\frac{y}{x}\right)+xy.\cos\left(\frac{y}{x}\right)+2y^2\cos\left(\frac{y}{x}\right)-\frac{y^3.sen\left(\frac{y}{x}\right)}{x}=2y^2\cos\left(\frac{y}{x}\right)+2x^2sen\left(\frac{y}{x}\right)=2[y^2\cos\left(\frac{y}{x}\right)+x^2sen\left(\frac{y}{x}\right)]=2z$

c) $x.z'_x+y.z'_y=x\cdot\frac{x}{x^2+y^2}+y\cdot\frac{y}{x^2+y^2}=\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}=\frac{x^2+y^2}{x^2+y^2}=1$
4) $Z'_y(0;4)=0$
5) a) $-\frac{\sqrt{6}}{3}$ b) $-\frac{3\sqrt{2}}{2}$

Diferenciales

1) a) $dz = (6x^2 - 4y^2)dx + (9y^2 - 8xy)dy$. b) $dz = 2x.\cos(2y)\,dx - 2x^2.sen(2y)dy$
c) $dz = [\ln(x+y) + \frac{x-y}{x+y}]dx + [\frac{x-y}{x+y} - \ln(x+y)]dy$. d) $du = y^2z^3dx + 2xyz^3dy + 3xy^2z^2dz$
2) $\Delta u = 91/4 = 22{,}75;\ du = 112/5 = 22{,}4$
3) $\Delta z = 99/1250\,;\ dz = 2/25$
4) $dz = 1$
5) $dz = -0{,}1516$
6) 0,9785
7) 2,01
8) La función no es diferenciable en el origen porque no es contínua.
9) $\varepsilon_a = 8{,}4.\ \varepsilon_{\%} = 0{,}75\%$.
10) Altura: $\varepsilon_a = \frac{\sqrt{3}}{15};\ \varepsilon_{\%} = 1{,}11\%$. Volumen: $\varepsilon_a = \frac{67+96\sqrt{3}}{10}\pi;\ \varepsilon_{\%} = 3{,}35\%$.
11) $-0{,}011$
12) $-41/900;\ -0{,}6212\%$
13) 0,00177
14) $16{,}31\,cm/seg^2$
15) $\frac{dt}{t} = \frac{dv}{v} + \frac{dp}{p}$
16) $-54{,}49\mathrm{Kg/m^2}$

Trabajo Práctico Nº 4: Funciones compuestas e implícitas

Funciones compuestas

1)51
2) 0
3) $\frac{\partial z}{\partial u} = 1001\,;\ \frac{\partial z}{\partial v} = 413$
4) $\frac{dz}{dt} = 2$
5) $\frac{dz}{dt} = -9$
6) $\frac{du}{dt} = 18$
7) $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \left[\left(lny + \frac{y}{x}\right).e^{u+v} + \left(\frac{x}{y} + lnx\right).e^{u-v}\right] + \left[\left(lny + \frac{y}{x}\right).e^{u+v} + \left(\frac{x}{y} + lnx\right).(-e^{u-v})\right] = 2\left(lny + \frac{y}{x}\right)e^{u+v}$
8) $Z'_u + Z'_v = \frac{y-x}{x^2+y^2} + \frac{x+y}{x^2+y^2} = \frac{2y}{x^2+y^2} = \frac{2(u-v)}{(u+v)^2+(u-v)^2} = \frac{2(u-v)}{u^2+2uv+v^2+u^2-2uv+v^2} = \frac{2(u-v)}{2(u^2+v^2)} = \frac{u-v}{u^2+v^2}$
9) -12
10) $40e^2 + 20e \approx 349{,}92788$

Funciones implícitas

1) a) $\frac{dy}{dx} = \frac{1}{y^4+y^2+1}$; b) $\frac{dy}{dx} = \frac{6\sqrt[3]{y^2}}{3\sqrt[6]{y}+2}$; c) $\frac{dy}{dx} = \frac{x^2-ay}{ax-y^2}$; d) $\frac{dy}{dx} = \frac{y}{x}$; e) $\frac{dy}{dx} = \frac{ctg(xy)}{x^2} - \frac{y}{x}$
2) a) $2^3 - (-2)^3 + 4.2.(-2) = 0;\ \frac{dy}{dx} = 1$; b) $2.2 - \sqrt{2.2.4} + 4 = 4;\ \frac{dy}{dx} = -2$; c) $e^0\cos(0+0) - 0 = 1;\ \frac{dy}{dx} = 1$ d) $2^2 + 2.3 + 2.3^2 = 28;\ \frac{dy}{dx} = \frac{-1}{2}$; e) $\sqrt{2.2} + \sqrt{3.3} = 5;\ \frac{dy}{dx} = -1$; f) $a^3 - a.a.a + 3aa^2 = 3a^3;\ \frac{dy}{dx} = -\frac{2}{5}$
3) a) $\frac{\partial z}{\partial x} = -1;\ \frac{\partial z}{\partial y} = -1$; b) $\frac{\partial z}{\partial x} = \frac{z}{1-2z};\ \frac{\partial z}{\partial y} = \frac{3z}{1-2z}$; c) $\frac{\partial z}{\partial x} = -\frac{z[2y.\cos(xyz)+3x^2]}{x[2y.\cos(xyz)+x^2]};\ \frac{\partial z}{\partial y} = \frac{3y^2-2xz.\cos(xyz)}{x[2y.\cos(xyz)+x^2]}$; d) $\frac{\partial z}{\partial x} = \frac{yz^2e^{xy-2}+2}{1-2ze^{xy-2}};\ \frac{\partial z}{\partial y} = \frac{xz^2e^{xy-2}-4}{1-2ze^{xy-2}}$
4) $\frac{dy}{dz} = -\frac{1}{4y};\ \frac{dz}{dy} = -4y$; b) $\frac{\partial u}{\partial x} = \frac{12v-1}{8uv-1};\ \frac{\partial v}{\partial x} = \frac{3-2u}{8uv-1};\ \frac{\partial u}{\partial y} = \frac{2(2v+1)}{8uv-1};\ \frac{\partial v}{\partial y} = \frac{4u+1}{8uv-1}$
5) $\frac{\partial u}{\partial x} = \frac{1-2ux}{x^2+y^2};\ \frac{\partial v}{\partial x} = -\frac{2vx}{x^2+y^2};\ \frac{\partial u}{\partial y} = -\frac{2uy}{x^2+y^2};\ \frac{\partial v}{\partial y} = -\frac{2vy+1}{x^2+y^2}$

Trabajo Práctico Nº 5: Derivadas y diferenciales sucesivas

Derivadas sucesivas

1) a) $Z''_{xy}=\frac{-2x}{(x^2+y^2)^2}=Z''_{yx}$; b) $Z''_{xy}=\frac{(ad-bc)(cx-dy)}{(cx+dy)^3}=Z''_{yx}$

2) a) $Z'_x=-\frac{y}{x^2+y^2}$; $Z'_y=\frac{x}{x^2+y^2}$; $Z''_{xx}=\frac{2xy}{(x^2+y^2)^2}$; $Z''_{xy}=\frac{y^2-x^2}{(x^2+y^2)^2}=Z''_{yx}$; $Z''_{yy}=-\frac{2xy}{(x^2+y^2)^2}$

b) $Z'_x=y^2x^{y^2-1}$; $Z'_y=2yx^{y^2}\ln(x)$; $Z''_{xx}=y^2x^{y^2-2}(y^2-1)$;

$Z''_{xy}=x^{y^2-1}[2y^3\ln(x)+2y]=Z''_{yx}$; $Z''_{yy}=x^{y^2}[4y^2ln^2(x)+2\ln(x)]$

c) $Z'_x=e^x\ln(y)+\frac{sen(y)}{x}$; $Z'_y=\frac{e^x}{y}+\ln(x).\cos(y)$; $Z''_{xx}=e^x\ln(y)-\frac{sen(y)}{x^2}$; $Z''_{xy}=\frac{e^x}{y}+\frac{cos(y)}{x}=Z''_{yx}$; $Z''_{yy}=-\frac{e^x}{y^2}-\ln(x)\,sen(y)$

d) $Z'_x=-\frac{sen[\ln(xy)]}{x}$; $Z'_y=-\frac{sen[\ln(xy)]}{y}$; $Z''_{xx}=\frac{sen[\ln(xy)]}{x^2}-\frac{cos[\ln(xy)]}{x^2}$;$Z''_{yy}=\frac{sen[\ln(xy)]}{y^2}-\frac{cos[\ln(xy)]}{y^2}$; $Z''_{xy}=-\frac{cos[\ln(xy)]}{xy}=Z''_{yx}$

3) a) $Z''_{xx}-4Z''_{yy}=[-4\cos(2x+y)-4sen(2x-y)]-4[-\cos(2x+y)-sen(2x-y)]=-4[\cos(2x+y)+sen(2x-y)]+4[\cos(2x+y)+sen(2x-y)]=0$

b) $Z''_{xx}+Z''_{yy}=[-e^{-t}sen(x)]+[-e^{-t}cos(y)]=-e^{-t}[sen(x)+cos(y)]=Z'_t$

Diferenciales Sucesivas

1) $d^2z=(48x^2y^2+2y^3)(dx)^2+[4xy(16x^2+3y)]dx.dy+(8x^4+6x^2y)(dy)^2$

2) $d^3z=\left[4xe^{x^2+y^2}(2x^2+3)\right](dx)^3+\left[12ye^{x^2+y^2}(2x^2+1)\right](dx)^2dy+\left[12xe^{x^2+y^2}(2y^2+1)\right]dx(dy)^2+\left[4ye^{x^2+y^2}(2y^2+3)\right](dy)^3$

Series de Taylor y Mac Laurin

1) a) $-\frac{16x^2+8\pi x(y-2)+\pi^2y^2-4\pi^2y+4\pi^2-8}{8}$; b) $\frac{2x(y+1)+y^2}{2}$

2) a) $\frac{e^2[x^3+3x^2(2y-1)+6x(2y^2-2y+1)+2(4y^3-6y^2+6y-1)]}{6}$; b) $-\left(x-\frac{\pi}{2}\right)-\left(y-\frac{\pi}{2}\right)+\frac{1}{6}[\left(x-\frac{\pi}{2}\right)^3+3\left(x-\frac{\pi}{2}\right)^2\left(y-\frac{\pi}{2}\right)+3\left(x-\frac{\pi}{2}\right)\left(y-\frac{\pi}{2}\right)^2+\left(y-\frac{\pi}{2}\right)^3]$

c) $-\frac{x^3}{6}+x-2y^2$; d) $\frac{y[3x^2+3x(2-y)+2y^2-3y+6]}{6}$

3) a) Entorno del origen:

$F(0;0)=0$; $F'_x(0;0)=0$; $F'_y(0;0)=1$; $F''_{xx}(0;0)=0$; $F''_{xy}(0;0)=\ln(a)$; $F''_{yy}(0;0)=-1$; $F'''_{xxx}(0;0)=0$; $F'''_{xxy}(0;0)=ln^2(a)$; $F'''_{xyy}(0;0)=-\ln(a)$; $F'''_{yyy}(0;0)=2$

$$z\approx F(0;0)+F'_x(0;0)x+F'_y(0;0)y+\frac{1}{2!}\left[F''_{xx}(0;0)x^2+2F''_{xy}(0;0)xy+F''_{yy}(0;0)y^2\right]$$
$$+\frac{1}{3!}\left[F'''_{xxx}(0;0)x^3+3F'''_{xxy}(0;0)x^2y+3F'''_{xyy}(0;0)xy^2+F'''_{yyy}(0;0)y^3\right]+\cdots$$
$$=0+0x+1y+\frac{1}{2!}\left[0x^2+2\ln(a)\,xy+(-1)y^2\right]+\frac{1}{3!}\{0x^3+3ln^2(a)x^2y$$
$$+3[-\ln(a)]xy^2+2y^3\}+\cdots$$
$$=y+\frac{1}{2}\left[2xyln(a)-y^2+x^2yln^2(a)-xy^2ln(a)\right]+\frac{1}{3}y^3+\cdots$$

b) Entorno del origen:

$F(0;0)=0$; $F'_x(0;0)=1$; $F'_y(0;0)=1$; $F''_{xx}(0;0)=0$; $F''_{xy}(0;0)=0$; $F''_{yy}(0;0)=0$; $F'''_{xxx}(0;0)=F'''_{xxy}(0;0)=F'''_{xyy}(0;0)=F'''_{yyy}(0;0)=-1$

$$z\approx 0+1x+1y+\frac{1}{2!}\left[0x^2+2.0xy+0y^2\right]+\frac{1}{3!}\left[-1x^3+3(-1)x^2y+3(-1)xy^2+(-1)y^3\right]$$
$$=x+y-\frac{\left(x^3+3x^2y+3xy^2+y^3\right)}{3!}+\cdots$$

Extremos Relativos

1) a) $\left(-\frac{4}{3};\frac{1}{3};-\frac{4}{3}\right)$ mínimo relativo; b) ∄ extremos relativos; c) $(-\frac{2}{3};\frac{1}{3};0)$ mínimo relativo; d) $(1;1;3)$ mínimo relativo; e) $(1;0;-2)$ mínimo relativo; $(-1;0;2)$ punto de ensilladura f) $(1;4;-19)$ mínimo relativo; g) $(0;0;5)$ máximo relativo; h) $(0;1;\frac{4}{3})$ máximo relativo; $(0;3;0)$ punto de

ensilladura; $(2;1;-\frac{20}{3})$ punto de ensilladura; $(2;3;-8)$ mínimo relativo; $\left(-5;1;-\frac{1109}{12}\right)$ punto de ensilladura; $(-5;3;-\frac{375}{4})$ mínimo relativo; i) $(0;0;0)$ mínimo relativo; j) $(-1;\frac{1}{2};\sqrt[4]{e^9})$ máximo relativo; k) $(1;0;0)$ mínimo relativo; l) $(0;0;0)$ punto de ensilladura; m) $\nexists$ extremos relativos; n) $(0;0;0)$ punto de ensilladura; $(\frac{3}{4};\frac{3}{8};-\frac{27}{32})$ mínimo relativo; o) $(\sqrt{2};\sqrt{3};-6\sqrt{3}-4\sqrt{2}+2)$ mínimo relativo; $(\sqrt{2};-\sqrt{3};6\sqrt{3}-4\sqrt{2}+2)$ punto de ensilladura; $(-\sqrt{2};\sqrt{3};-6\sqrt{3}+4\sqrt{2}+2)$ punto de ensilladura; $(-\sqrt{2};-\sqrt{3};6\sqrt{3}+4\sqrt{2}+2)$ máximo relativo.

2) $k=2$, mínimo relativo.

3) $k=3$, mínimo relativo.

Trabajo Práctico Nº 6: Integrales paramétricas

1) a) $y^2(e-1)$; b) $-6arctg\sqrt{y^2-16}$; c) x^2; d) $\frac{\pi\sqrt{y}}{2}$; e) $2\sqrt{3x}\cdot arctg\left(\frac{\sqrt{3x}}{3}\right)+x.\ln(x^2+3x)-2x$

2) a) $\frac{1}{\sqrt{x^3}}+\frac{1}{2}$; b) $2y$

3) a) $\frac{\pi a^2}{2}$; b) $Ln(\frac{9}{8})$; c) 1

Trabajo Práctico Nº 7: Integrales Múltiples

1) $\frac{\pi}{2}$

2) a) $\frac{b^2}{3}$; b) $\sqrt{2}-1$; c) $\frac{1}{10}$ d) 2π

3) $2a^2(\frac{\pi}{3}+\sqrt{3})$

4) a) $\frac{abc}{6}$; b) $\frac{b^3\pi}{4}-\frac{b^3}{3}$; c) 36π

5) $x_G=2$; $y_G=3$; $M_x=\frac{27}{2}$; $M_y=9$

6) $\frac{4}{3}(8-3\sqrt{3})a^3\pi$

7) a) $I_x=\frac{63}{20}$; $I_y=\frac{729}{140}$; b) $I_x=\frac{1}{28}$; $I_y=\frac{1}{20}$

8) $z_G=\frac{15}{8}$; por simetría: $x_G=y_G=0$

9) $\frac{96\pi}{5}$; 10) $\frac{64\pi}{3}$

Trabajo Práctico Nº 8: Geometría diferencial

1) a) $6\vec{\imath}+5\vec{\jmath}-5\vec{k}$; $7\vec{\imath}-10\vec{\jmath}+10\vec{k}$; b) -2; 7; -5; c) $-1\vec{\imath}+13\vec{\jmath}+8\vec{k}$; $-4\vec{\imath}-5\vec{\jmath}-6\vec{k}$; d) -19

2) a) $\frac{dr}{dt}=4\vec{\imath}+3\vec{\jmath}-e\vec{k}$; $\frac{d^2r}{dt^2}=4\vec{\imath}+6\vec{\jmath}-e\vec{k}$; $\left|\frac{dr}{dt}\right|=\sqrt{e^2+25}$; $\left|\frac{d^2r}{dt^2}\right|=\sqrt{e^2+52}$

b) $\frac{dr}{dt}=-2\vec{\imath}-3\vec{k}$; $\frac{d^2r}{dt^2}=-3\vec{\jmath}$; $\left|\frac{dr}{dt}\right|=\sqrt{13}$; $\left|\frac{d^2r}{dt^2}\right|=3$

3) a) $\vec{A}=a_1(t)\vec{\imath}+a_2(t)\vec{\jmath}+a_3(t)\vec{k}$; $\vec{B}=b_1(t)\vec{\imath}+b_2(t)\vec{\jmath}+b_3(t)\vec{k}$

$$\frac{d}{dt}(\vec{A}\cdot\vec{B})=\frac{d}{dt}\{[a_1(t)\vec{\imath}+a_2(t)\vec{\jmath}+a_3(t)\vec{k}]\cdot[b_1(t)\vec{\imath}+b_2(t)\vec{\jmath}+b_3(t)\vec{k}]\}$$

$$=\frac{d}{dt}[a_1(t)\cdot b_1(t)+a_2(t)\cdot b_2(t)+a_3(t)\cdot b_3(t)]$$

$$=\frac{d}{dt}[a_1(t)\cdot b_1(t)]+\frac{d}{dt}[a_2(t)\cdot b_2(t)]+\frac{d}{dt}[a_3(t)\cdot b_3(t)]$$

$$=a_1'(t)\cdot b_1(t)+a_1(t)\cdot b_1'(t)+a_2'(t)\cdot b_2(t)+a_2(t)\cdot b_2'(t)+a_3'(t)\cdot b_3(t)+a_3(t)\cdot b_3'(t)$$

$$=a_1'(t)\cdot b_1(t)+a_2'(t)\cdot b_2(t)+a_3'(t)\cdot b_3(t)+a_1(t)\cdot b_1'(t)+a_2(t)\cdot b_2'(t)+a_3(t)\cdot b_3'(t)=\frac{d}{dt}(\vec{A})\cdot\vec{B}+\vec{A}\cdot\frac{d}{dt}(\vec{B})$$

b) $\vec{A} = a_1(t)\vec{\imath} + a_2(t)\vec{\jmath} + a_3(t)\vec{k}$; ϕ: función escalar

$$\frac{d}{dt}(\phi\vec{A}) = \frac{d}{dt}\left[\phi a_1(t)\vec{\imath} + \phi a_2(t)\vec{\jmath} + \phi a_3(t)\vec{k}\right]$$
$$= [\phi' a_1(t) + \phi a_1'(t)]\vec{\imath} + [\phi' a_2(t) + \phi a_2'(t)]\vec{\jmath} + [\phi' a_3(t) + \phi a_3'(t)]\vec{k}$$
$$= \phi a_1'(t)\vec{\imath} + \phi a_2'(t)\vec{\jmath} + \phi a_3'(t)\vec{k} + \phi' a_1(t)\vec{\imath} + \phi' a_2(t)\vec{\jmath} + \phi' a_3(t)\vec{k}$$
$$= \phi\left[a_1'(t)\vec{\imath} + a_2'(t)\vec{\jmath} + a_3'(t)\vec{k}\right] + \left[a_1(t)\vec{\imath} + a_2(t)\vec{\jmath} + a_3(t)\vec{k}\right]\phi'$$
$$= \phi\frac{d}{dt}(\vec{A}) + \vec{A}\frac{d}{dt}(\phi)$$

4) $|\vec{v}| = 0$; $|\vec{a}| = 12\sqrt{12}$

5) $\frac{6\sqrt{14}}{7}$; $-\frac{\sqrt{14}}{7}$

6) a) $-\vec{\jmath} + \vec{k}$; b) $\frac{-\sqrt{2}}{2}\vec{\jmath} + \frac{\sqrt{2}}{2}\vec{k}$; c) Ec. recta tangente: $\bar{Y} = \vec{\imath} - \frac{\sqrt{2}}{2}t\vec{\jmath} + (\frac{\sqrt{2}t}{2} + \frac{\pi}{2})\vec{k}$; d) $\bar{K}\left(\frac{\pi}{2}\right) = -\frac{1}{2}\vec{\imath}$; $R = 2$; e) $\bar{N}\left(\frac{\pi}{2}\right) = -\vec{\imath}$; Ec. la recta normal: $(1-t)\vec{\imath} + \frac{\pi}{2}\vec{k}$; f) $\bar{B}\left(\frac{\pi}{2}\right) = -\frac{\sqrt{2}}{2}\vec{\jmath} - \frac{\sqrt{2}}{2}\vec{k}$; Ec. la recta binormal: $\vec{\imath} - \frac{\sqrt{2}}{2}t\vec{\jmath} + (\frac{\pi}{2} - \frac{\sqrt{2}t}{2})\vec{k}$; g) Ec. plano normal: $-\frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2}z - \frac{\pi\sqrt{2}}{4} = 0$; Ec. plano rectificante: $-x + 1 = 0$; Ec. plano osculador: $-\frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}z + \frac{\pi\sqrt{2}}{4} = 0$

7) $3\sqrt{2}$

Trabajo Práctico Nº 9: Campos escalares y vectoriales

1) $10\vec{\imath} - 4\vec{\jmath} - 16\vec{k}$

2) $-\frac{x}{r^3}\vec{\imath} - \frac{y}{r^3}\vec{\jmath} - \frac{z}{r^3}\vec{k}$

3) $-\frac{1}{3}\vec{\imath} + \frac{2}{3}\vec{\jmath} + \frac{2}{3}\vec{k}$

4) $-2x + y + 3z - 1 = 0$

5) $\frac{376}{7}$

6) $-\frac{20}{9}$

7) $2x\vec{\jmath}$

8) a) $2x^3yz^3(2z - x)$; b) $-2x^4yz^3 + 6x^3yz^4 + 8xy^2z^4$

c) $2yz(2x + z)\vec{\imath} - x^2y(2x + z)\vec{\jmath} + xz^2(2x + z)\vec{k}$

d) $2x^2yz^3(2y - z^2)\vec{\jmath} - 4x^2yz^3(xy + z)\vec{k}$

e) $(-6x^4y^2z^2 - 2x^3z^5)\vec{\imath} + x^2(4yz^5 - 12y^2z^3)\vec{\jmath} + (4x^3y^2z^3 + 4x^2yz^4)\vec{k}$

9) $-7/3$

10) $8x + 8y - z - 12 = 0$

11) 0

Apéndice elaborado por Ing. Manuel Zeniquel – 2013.